

# The stability of a horizontal fluid interface in a vertical electric field

By G. I. TAYLOR AND A. D. MCEWAN

Cavendish Laboratory, Cambridge

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The stability of the horizontal interface between conducting and non-conducting fluids under the influence of an initially uniform vertical electric field is discussed. To produce such a field when the conducting fluid is the heavier it is imagined that a large horizontal electrode is immersed in the non-conducting fluid. As the field increases the part of the interface below the electrode rises till at a voltage  $V$ , which depends on the interfacial tension, the height of the electrode above the interface and the density difference, the interface becomes unstable for vertical displacements  $Z$  which satisfy the equation

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right) Z = 0.$$

The value of  $k$  consistent with the lowest value of  $V$  is found. When the electrode is situated above the interface at less than a certain distance the lowest value of  $V$  is attained when  $k = 0$  so that the horizontal extent of an unstable crest is likely to be great. As the electrode height increases above this critical value  $k$  increases and the unstable crests become more closely spaced till an upper limiting value of  $k$  is obtained.

Experiments made with several pairs of fluids verify these theoretical conclusions. In some cases sparking occurs before the potential  $V$  is reached, but in others, air at atmospheric pressure over water, for instance, the instability occurs first and the jet of water which results permits the passage of a spark which may inhibit further development of the instability. The physical condition which determines the sparking voltage to a fluid may therefore be very different from that which is operative between solid electrodes. This consideration might be relevant to the performance of power-line insulators in wet weather.

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## 1. Introduction

In discussing the disintegration of water drops in a strong electric field (Taylor 1964) it was shown that the internal pressure, though constant inside the drop, has an important effect on its stability. An analogous effect must occur at the horizontal interface between a conducting fluid such as water and a lighter non-conducting fluid when a strong vertical electric field is applied. A local vertical displacement of the interface will concentrate the lines of force and so increase the vertical force on the interface. If this increase is great enough to counterbalance the pressure drop due to gravity and the surface tension, the interface

will be unstable. The physical conditions under which this instability may occur can be realized by supporting a horizontal conducting plate at height  $H$  above the interface and raising it to potential  $V_0$ . The value of  $V_0$  at which the interface is in neutral equilibrium can be calculated by straightforward methods.

Suppose that the potential of the conducting plate is gradually raised from zero. The water surface under it will rise and, if the vessel containing it is of finite extent, the parts of the surface which are outside the influence of the plate will fall. The ratio of the rise under the plate to the fall at distant points will depend on the ratio of the area of the plate to that of the container in which the experiment is performed. If  $h$  is the height of the plate above the raised interface when neutral conditions are established and  $h_0$  is its height above the level of the interface at points outside the range of the electric field the decrease in pressure under the raised interface due to gravity is  $(\rho_1 - \rho_2)(h_0 - h)g$ , where  $\rho_2$  and  $\rho_1$  are the densities of the upper and lower fluids respectively. In the following analysis it is assumed that the upper electrode radius is much greater than its height above the conducting fluid.

If the interface is displaced vertically through a small height

$$Z = Bf(x, y) \quad (1)$$

from the level to which it had been raised by the field under the upper electrode, the equation for neutral equilibrium is satisfied when the surface stress due to the electric field is

$$g(\rho_1 - \rho_2)\{h_0 - h + Bf(x, y)\} + T\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \quad (2)$$

Here  $r_1$  and  $r_2$  are the principal radii of curvature of the displaced interface and  $T$  is the interfacial tension. Since  $B$  will be assumed small compared with the lateral extent of disturbances to be considered

$$\frac{1}{r_1} + \frac{1}{r_2} = -B\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)f(x, y). \quad (3)$$

If the potential of the interface is zero, that of the upper electrode is  $V_0$  and the vertical co-ordinate  $z$  is measured from the level to which the lower fluid is raised when the neutral equilibrium is attained, a possible potential field which satisfies the necessary electrical boundary condition  $V = V_0$  at  $z = h$  is

$$V = V_0\left\{\frac{z}{h} + \frac{\sinh k(z-h)}{\sinh kh} Bf(x, y)\right\}, \quad (4)$$

and  $V$  is a potential function provided

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0. \quad (5)$$

The component of stress normal to the interface is

$$\frac{K}{8\pi}\left\{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right\},$$

where  $K$  is the dielectric constant of the non-conducting fluid. Since the slope of the interface is assumed to be small, only  $K(\partial V/\partial z)^2/8\pi$  need be taken into account in the linearized equation of neutral equilibrium, which is

$$\frac{K}{8\pi} \frac{V_0^2}{h^2} \{1 + 2kB \coth(kh) \cdot f(x, y)\} = g(\rho_1 - \rho_2) \{h_0 - h + Bf(x, y)\} + BTk^2 f(x, y), \quad (6)$$

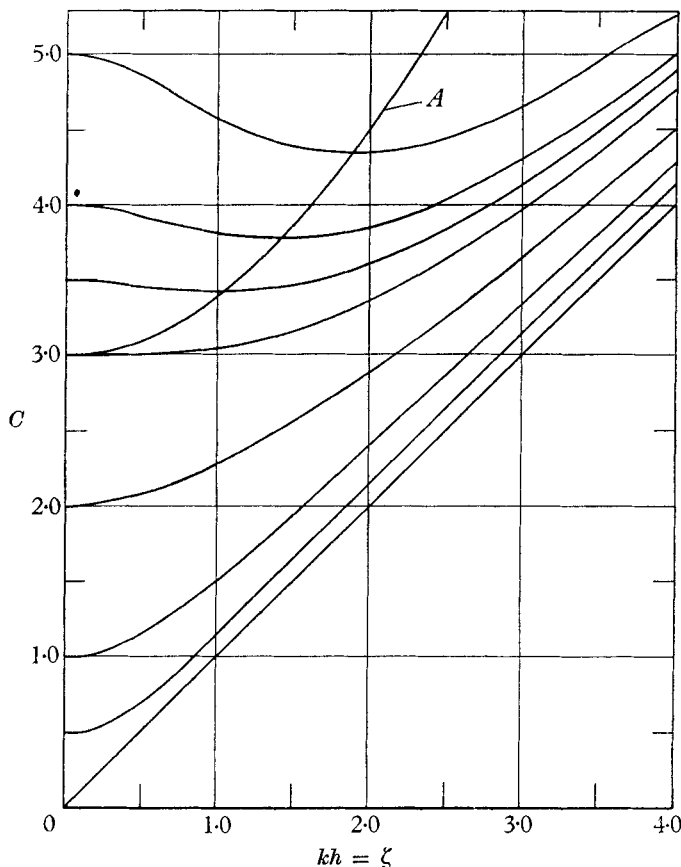


FIGURE 1. Variation of  $C$  with  $\zeta$  for constant  $h'^2$ .  $h'^2 = C$  when  $\zeta = 0$ .  
Curve  $A$  is  $C = 4\zeta \sinh^2 \zeta / (\sinh 2\zeta - 2\zeta)$ .

and (6) is satisfied provided

$$\frac{KV_0^2}{8\pi h^2} = g(\rho_1 - \rho_2)(h_0 - h), \quad (7)$$

and

$$\frac{KV_0^2}{4\pi h^2} = \tanh kh \left\{ Tk + g \frac{(\rho_1 - \rho_2)}{k} \right\}. \quad (8)$$

These can be expressed in non-dimensional form. Writing

$$h' = hg^{\frac{1}{2}}(\rho_1 - \rho_2)^{\frac{1}{2}}T^{-\frac{1}{2}}, \quad k' = kT^{\frac{1}{2}}(\rho_1 - \rho_2)^{-\frac{1}{2}}g^{-\frac{1}{2}}, \quad C = KV_0^2/4\pi Th \quad \text{and} \quad \zeta = kh = k'h', \quad (9)$$

(7) becomes  $h_0/h = 1 + C/2h'^2$ , (10)

and (8) becomes  $C = \frac{\tanh \zeta}{\zeta} (\zeta^2 + h'^2)$ . (11)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
$Kh$	$h'$	$C$	$h/h_0$	$(h'C)^{\frac{1}{2}}$	$h_0$ (cm)	$V_0$ (kV)	$h_0$ (cm)	$V_0$ (kV)	$h_0$ (cm)	$V_0$ (kV)	$h$ (cm)	$h_0$ (cm)	$V_0$ (kV)
0	0.4	0.160	1.500	0.213	0.164	1.125	0.340	0.780	0.283	0.36	0.062	0.93	1.16
0	0.8	0.640	1.500	0.715	0.3274	3.393	0.681	2.354	0.566	1.07	0.124	1.86	3.50
0	1.2	1.440	1.500	1.314	0.491	6.236	1.020	4.327	0.850	1.99	0.186	0.279	6.40
0	1.6	2.560	1.500	2.024	0.655	9.606	1.360	6.665	1.13	3.07	0.248	0.372	9.90
0	1.732	3.000	1.500	2.279	0.709	10.816	1.473	7.505	1.226	3.46	0.268	0.403	11.16
0.6	1.775	3.143	1.499	2.362	0.726	11.210	1.501	7.778	1.257	3.58	0.276	0.414	11.58
1.0	1.860	3.395	1.491	2.513	0.756	11.927	1.571	8.275	1.309	3.82	0.288	0.430	12.30
1.5	2.043	3.875	1.464	2.813	0.816	13.350	1.695	9.263	1.412	4.27	0.316	0.461	13.75
2.0	2.319	4.518	1.420	3.237	0.898	15.363	1.867	10.659	1.551	4.91	0.359	0.510	15.8
2.5	2.675	5.289	1.369	3.761	0.999	17.850	2.077	12.385	1.728	5.71	0.414	0.566	18.4
3.0	3.096	6.154	1.321	4.365	1.116	20.716	2.326	14.374	1.930	6.62	0.480	0.634	21.4
3.5	3.544	7.078	1.282	5.008	1.239	23.768	2.575	16.491	2.144	7.60	0.548	0.700	24.5
4.0	4.021	8.040	1.248	5.686	1.369	26.986	2.845	18.724	2.369	8.63	0.622	0.774	27.8

Columns 1-5, non-dimensional; 6-7, air-water; 8-9, oil-water; 10-11, oil-alcohol water mixture; 12-14 mercury-oil.

TABLE 1

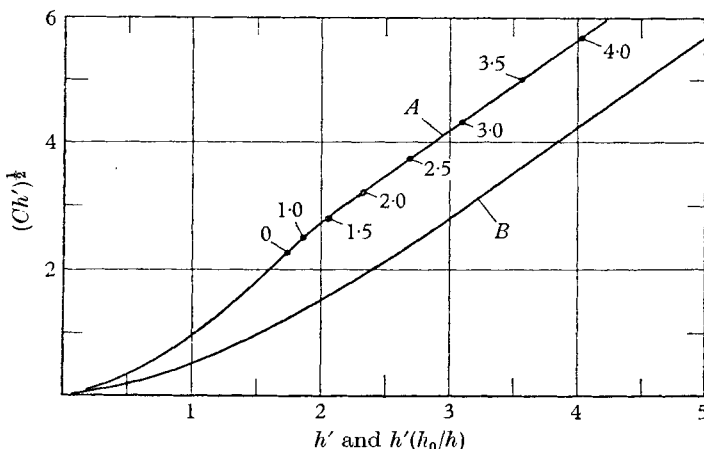


FIGURE 2. Minimum values of  $(Ch')^{\frac{1}{2}}$  and corresponding values of  $h'$  (curve A), and  $h'(h_0/h)$  (curve B). Numbers are values of  $kh$ .

In particular, when  $\zeta = 0$ ,  $C = h'^2$ . Figure 1 shows how  $C$  varies with  $\zeta$  when  $h'$  is constant. It will be noticed that when  $h' > \sqrt{3}$  each curve has a minimum but when  $h' < \sqrt{3}$  the least value of  $C$  occurs when  $\zeta = 0$ . These curves may be regarded as the curves of neutral stability.

For a given value of  $h'$ , the value of  $V_0$  at which instability first occurs when  $V_0$  increases is that for which  $C$  is a minimum as  $\zeta$  varies. This condition gives

$$C = \frac{4\zeta \sinh^2 \zeta}{\sinh 2\zeta - 2\zeta} \quad \text{and} \quad h'^2 = \zeta^2 \frac{\sinh 2\zeta + 2\zeta}{\sinh 2\zeta - 2\zeta}. \quad (12)$$

The data which can conveniently be measured experimentally are  $V_0$  and  $h_0$ , and from (9)

$$h = h' \left[ \frac{T}{g(\rho_1 - \rho_2)} \right]^{\frac{1}{2}}, \quad (13)$$

$$V_0 = C^{\frac{1}{2}} h'^{\frac{1}{2}} \left( \frac{4\pi}{K} \right)^{\frac{1}{2}} \left( \frac{T}{g(\rho_1 - \rho_2)} \right)^{\frac{1}{4}} T^{\frac{1}{2}}, \quad (14)$$

$$\left. \begin{aligned} h_0 &= h \left( 1 + \frac{C}{2h'^2} \right) \quad \text{when } h' > \sqrt{3}, \\ h_0 &= \frac{3}{2}h \quad \text{when } h' < \sqrt{3}. \end{aligned} \right\} \quad (15)$$

For experiments in which  $V_0$  is raised till instability occurs it is therefore convenient to tabulate the value of  $C^{\frac{1}{2}}h'^{\frac{1}{2}}$  corresponding with the minimum value of  $C$  when  $h'$  is fixed. When  $h' > \sqrt{3}$  this can be done by calculating  $C$  and  $h'$  for arbitrarily chosen values of  $\zeta$  using (12). When  $h' < \sqrt{3}$ ,  $C^{\frac{1}{2}}h'^{\frac{1}{2}} = h^{\frac{3}{2}}$ . Some values are given in column 5 of table 1 and are shown in figure 2.

## 2. Physical character of disturbances

The small disturbances which occur when  $V_0$  reaches the critical value are characterized only by the value of  $k$  corresponding with the minimum  $V_0$ . Any disturbance which satisfies (5) is then possible. To obtain a unique solution which would define the disturbances more exactly it would be necessary to find boundary conditions to be satisfied at the edge of the disturbed area. It has been assumed, however, that  $h_0$  is small compared with the linear dimensions of the charged plate and under these conditions the distant boundary must have little influence in determining the form of the disturbances. The only limitation to the geometrical character of the disturbed surface is contained in (5). The practical effect of this limitation seems to be to define roughly the distance of a crest or summit from its nearest neighbours. Thus, for instance, in the regularly spaced disturbances represented by

$$f = \sin kx \quad \text{or} \quad f = \sin \frac{kx}{\sqrt{2}} \sin \frac{ky}{\sqrt{2}},$$

neighbouring summits are at distance  $2\pi/k$  apart, while for Christopherson's (1940) hexagonal distribution,

$$f = 2 \cos \frac{1}{2}\sqrt{3} kx \cos \frac{1}{2}ky + \cos ky,$$

this distance is  $4\pi/\sqrt{3}k$ . The linear theory does not seem capable of distinguishing between them. Analogous conditions occur in the cases of instability due to heating a fluid from below and of the instability of a horizontal free surface subjected to vertical oscillations.

### *Limiting conditions*

When  $h' < \sqrt{3}$ ,  $kh = 0$ ,  $C = h'^2$ , or

$$V_0 = \left[ \frac{4\pi g(\rho_1 - \rho_2)}{K} \right]^{\frac{1}{2}} h^{\frac{3}{2}}, \quad (16)$$

and (15) becomes 
$$h_0 = \frac{3}{2}h. \quad (17)$$

The significance of this expression can be appreciated by assuming from the outset that the interface remains plane and is disturbed only by a vertical displacement. The equation of equilibrium is then

$$KV_0^2/8\pi h^2 = g(\rho_1 - \rho_2)(h_0 - h). \quad (18)$$

The maximum value of  $V_0$  for which (18) can be satisfied corresponds with the maximum value of  $h^2(h_0 - h)$  as  $h$  varies while  $h_0$  remains constant. This maximum is  $\frac{4}{27}h_0^3$  or  $\frac{1}{2}h^3$  when  $h = \frac{2}{3}h_0$ . When this value is inserted in (18), (16) is recovered. *kh large.* When  $kh$  is large  $\tanh kh \sim 1.0$  so that (8) becomes

$$\frac{V_0}{h} = \left(\frac{4\pi}{K}\right)^{\frac{1}{2}} \left\{ Tk + \frac{g(\rho_1 - \rho_2)}{k} \right\}^{\frac{1}{2}}. \quad (19)$$

The minimum value occurs when  $k^2 = g(\rho_1 - \rho_2)/T$  and is

$$\frac{V_0}{h} = \left(\frac{8\pi}{K}\right)^{\frac{1}{2}} \{gT(\rho_1 - \rho_2)\}^{\frac{1}{4}}. \quad (20)$$

If the disturbance is in the form of plane waves, their length is

$$\lambda = 2\pi(T/[g(\rho_1 - \rho_2)])^{\frac{1}{2}}.$$

It is of interest to note that when  $\rho_2 = 0$  this is the length of the surface wave which travels with least velocity on deep water.

### 3. Comparison with observation

Experiments were made in which a horizontal charged metal disk 12.7 cm in diameter was supported at various heights  $H$  above the conducting fluid which was water, dilute alcohol or mercury. The conducting fluid was contained in a tank with transparent walls so that its surface could be photographed. Figure 3 is a sketch of the apparatus. The height  $H$  of the electrode  $E_1$  above the water surface was measured before charging it to potential  $V_0$ . When the potential gradient was established the level of the water under the electrode rose and that at distant points fell by an amount  $\delta h$  which was measured.

The object of the experiment was to observe the form of the instability, and to obtain data for a comparison with the calculated relation between  $V_0$  and  $h$ , the distance between the electrode  $E_1$  and the liquid interface at the onset of instability.

It was found that when the liquid interface became unstable, agitation of the surface made a direct measurement of  $h$  impossible. Instead the fall,  $\delta h$ , in level of the interface at a point remote from  $E_1$ , was measured, and by measuring the height  $H$  of  $E_1$  above the interface in the absence of an applied voltage, the value of  $h_0$  could be estimated closely, as  $H + \delta h$ .

The source of high potential, kindly lent to us by Prof. C.W. Oatley, was capable of generating up to 25 kV d.c. Readings on a kilovoltmeter  $KV$  and micro-ammeter  $MA$  in series with a resistance of  $2 \times 10^9$  ohm (figure 3) enabled the mean voltage between  $E_1$  and  $E_2$  to be measured. The current through the

micro-ammeter became appreciable as the interface became unstable owing to the formation of conduction paths in the non-conductor by the conducting liquid which were of appreciable conductivity. The period of time for which  $E_1$  and  $E_2$  were actually short circuited in this way was estimated using a cathode-ray

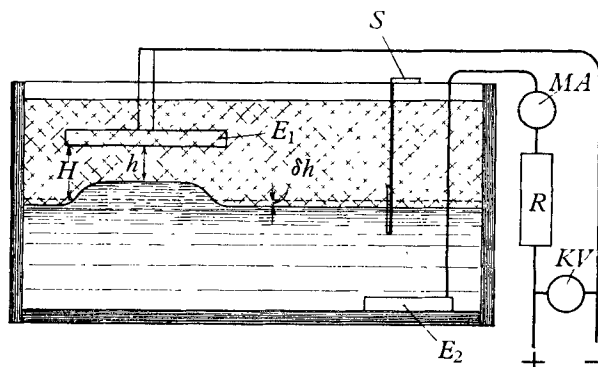


FIGURE 3. Apparatus.

oscilloscope during experiments with the water-air interface, and was found to form a minute proportion of the time between successive short circuits. This is in agreement with the observation (Taylor 1964) that in a 50 ft. length of a cinema record of this instability only one frame showed a jet in process of formation.

Though the time of existence of short circuiting paths was very small it was found that the mean conductivity produced in the non-conducting fluid as a result of the instability raised the potential recorded on  $KV$  (figure 3), but when  $V$  was corrected using the appropriate formula,

$$V_0 = \text{reading on } KV - 2 \times (\text{reading on } MA), \quad (22)$$

the resulting values of  $V_0$  were remarkably constant, and agreed with the value observed when the instability first made its appearance.

When the experiments were made with air and water it was noticed that directly the instability started sparks occurred and it was thought that possibly the disturbed state of the water surface was due to the occurrence of cascading ionization by collision rather than to instability. Experiments were therefore performed with oil as the non-conducting fluid. This had the advantage that electrical breakdown of the oil was unlikely and also that the dielectric constant of oil is more than twice as great as that of air so that the voltage necessary for instability is correspondingly reduced. Later, experiments were made in which the lower fluid was a mixture of equal volumes of alcohol and water. Experiments were also made with mercury under air and under oil.

#### *Physical data*

The oil used was Shell Diala which is known as 'transformer' oil. Its density was found to be  $0.882 \text{ g cm}^{-3}$  and the dielectric constant was taken to be 2.2 which is the value given by the suppliers (and also in several physical tables).

The density of the dilute alcohol was  $0.924 \text{ g cm}^{-3}$ . The surface tension of the oil-water and the oil-dilute alcohol interfaces were measured by the hanging drop method using the computations of Fordham (1948).

The interfacial tension of the mercury-air and mercury-oil interfaces were found in two ways (i) by measuring the depth of a large mercury drop on a horizontal plane (Quincke's method); and (ii) by measuring the depth of the interface in a 1 mm diameter glass tube below a large interface into which the tube was thrust. The second method involved inserting a fine iron wire into the upper part of the capillary tube. Two other iron wires were mounted on a Perspex carrier which could slide on the tube. These two wires were connected together and when either of them touched the mercury they completed a circuit which

1	2	3	4	5	6
	$\rho_1 - \rho_2$ ( $\text{g cm}^{-3}$ )	$T$ ( $\text{dyn cm}^{-1}$ )	$h/h'$ (cm)	$V_0/C^{\frac{1}{2}}h^{\frac{1}{2}}$ (kV)	limiting gradient ( $\text{kV cm}^{-1}$ )
Water-air	1.0	73	0.273	4.75	47.2
Water-oil	0.118	37.2	0.567	3.29	8.2
50/50 water alcohol-oil	0.042	9.3	0.469	1.40	4.5
Mercury-air	13.6	(i) 356 (ii) 350	0.162	5.4	70.2
Mercury-oil	12.7	(i) 295 (ii) 303	0.155	4.89	44.6

Instability data for different interfaces. Surface tensions (column 3) are as given (i) by Quincke's method and (ii) by glass capillary tube method.

In column (6), the limiting gradient is  $0.3\{gT(\rho_1 - \rho_2)\}^{\frac{1}{2}}(8\pi/K)^{\frac{1}{2}}$ .

TABLE 2.

showed the instant of contact. The wire inside the capillary tube was also connected with another voltmeter which showed when the mercury inside the tube had risen to the lower end of the inner wire. To make a measurement the position of the Perspex carrier on the capillary tube was adjusted till on inserting the tube into the mercury through the air or oil which lay above it the contacts inside and outside the tube were made simultaneously. The vertical distance between the iron wires inside and outside the tube was then the hydrostatic head which was balanced against the interfacial tension.

To make accurate measurements by either of the methods (i) or (ii) it would be necessary to know the angle of contact, but with mercury and Perspex or glass this angle was near enough to  $180^\circ$  to make the error due to assuming it to be  $180^\circ$  tolerable in view of other uncertainties in the experiment.

The data used in comparing the observed with the calculated conditions for instability are given in columns 2 and 3 of table 2 and the factors which enable the corresponding values of  $h$  and  $V_0$  to be found are given in columns 4 and 5. Column 6 gives the limiting electric intensity in kilovolts per cm at which a horizontal interface becomes unstable as the gap between plate and interface increases. It should be noticed that potentials in the theoretical formula are in electrostatic units and to convert them to kilovolts these must be multiplied by a factor 0.3.



*Experimental results*

As the critical voltage for instability was approached the interface became rather agitated and occasional isolated points would rise from the lower (conducting) fluid. These sometimes appeared to be connected with bubbles of the water in the oil or detached drops of water on the water surface when the non-conducting fluid was air. Sometimes they were connected with visible specks of dust.

At this stage a small increase in  $V$  gave rise to violent agitation of the interface. Pointed conical disturbances rose and fell in many places over the interface and jets of water penetrated into the upper fluid. Although it was not always easy to associate exactly a definite visible amount of disturbance with the theoretical neutral stability the scatter of readings of  $V_0$  was not large when the correction formula (22) was used, even when there was a large scatter in the readings of the voltmeter kV of figure 3. When the upper fluid was oil it was found that after the apparatus had operated for a short time the surface of the lower fluid appeared to become slightly clouded. A scraper  $S$  (figure 3) similar to that used in Langmuir's experiments on surface films was therefore arranged so that it could be pulled the length of the trough. It was found that a surface film was produced whenever a number of jet-like disturbances had passed through the oil. The conductivity of the oil rose and a current of several micro-amperes might occur before neutral equilibrium was attained. The normal non-conducting character of the oil was restored by a passage of the scraper along the trough, and before each recorded observation this was done.

*Air-water interfaces*

The voltage at which the surface first became disturbed and sparking began was very clearly defined and repeatable, but immediately this happened the voltage between the electrodes dropped owing apparently to the formation of an un-

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$h_0$ (cm)	0.226	0.336	0.440	0.556	0.660	0.762	0.862	0.962	1.062	1.166	1.270
$V_0$ (kV)	1.6	3.3	5.3	7.2	9.8	12.1	14.6	17.3	19.6	20.5	25.1

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TABLE 3. Observed voltages for instability at a water-air interface

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broken jet of water which immediately lowered the resistance between the electrodes. When this occurred the potential drop between the electrodes could not usually be raised to the value at which the jet first appeared even though the voltage of the supply was much greater. In fact the drop in potential at the resistance  $R = 2 \times 10^9$  ohm (shown in figure 3) increased faster than the rise in voltage applied by the generator. The measurements of  $V_0$  and  $h_0$  are given in table 3. The calculated values using the data of table 2 are given in columns 6 and 7 of table 1, and are plotted in the full curve of figure 4. It will be seen that the agreement is extremely good. The conclusion which must be drawn from this is that when sparking occurs in these experiments at a water surface the physical cause of the phenomenon lies in the behaviour of the water surface rather than in ionization by collision or other processes of breakdown in the air. To test this

conclusion, experiments were made in which the water was replaced by a metal (brass) and the sparking potential measured. Since the general level of the water under the upper electrode had been raised by the electric field before instability and sparking occurred the lower electrode was made in the form of a disk of the same size as the upper one. In the first of these experiments neither the upper

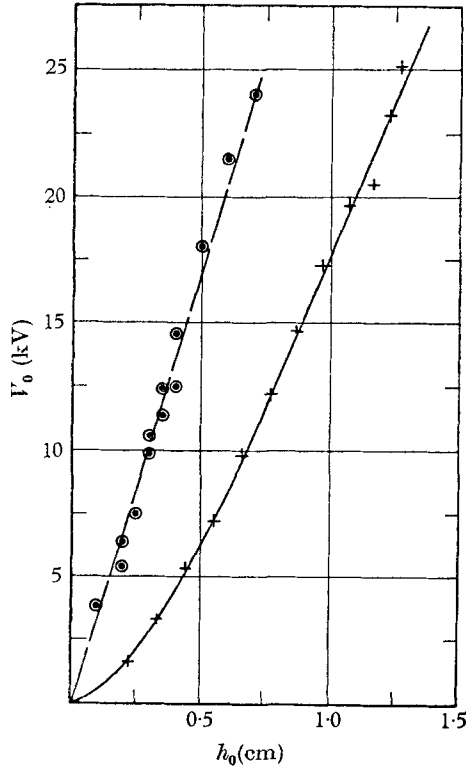


FIGURE 4. Sparking potentials in air between horizontal electrodes. Experimental points: + lower electrode water,  $\odot$  lower electrode brass. Full curve calculated instability for water; calculated limiting slope 24.6 kV/cm. Broken line 34.0 kV/cm.

nor the lower electrode had well-rounded edges and sparks always occurred between the two edges. When the upper electrode which was 0.37 cm thick was rounded so that its edge section was a semicircle and the lower electrode was rounded on the upper side at the edge to a larger radius, the sparks appeared at random all over the flat central parts of the lower electrode. The results of these measurements are also shown in figure 4. Apart from a small scatter they show a linear relationship between sparking voltage and  $h_B$ , the distance between the brass plates corresponding to 34.0 kV/cm.

The sparking voltages to water and to brass could be compared on the basis that either  $h_0$  or  $h$  is comparable with  $h_B$ . Since  $h_0$  is the quantity which could be measured figure 4 is constructed to reveal the comparison between  $h_0$  and  $h_B$ . It will be seen that  $h_B$  is always greater than  $h_0$  for a given sparking voltage. It was found that if the values of  $h$  corresponding to any given value of  $h_0$  were calculated from equation (15) the water-air-brass sparking curve is still below

the brass-air-brass curve. The difference between  $h$  and  $h_0$  is only significant at small values of  $h'$  and for sparks from water through air the difference between  $h_0$  and  $h$  is never greater than 0.273 cm. At distances greater than 1 cm the potential gradient for instability at the water surface is

$$V_0/h_0 = 300(8\pi)^{\frac{1}{2}} \{(981)(73)\}^{\frac{1}{2}} = 24.6 \text{ kV/cm}, \quad (23)$$

which is less than 34.0 kV/cm. This explains why in our experiments with air over water instability preceded sparking. However, the potential gradient for very large spark gaps in air at atmospheric pressure is less than 34 kV/cm so that it cannot be stated as a general proposition that instability could always occur before dielectric breakdown.

#### Experiments with oil over water

Though the instability appears to be the physical cause which starts sparking from a metal plate to water through air, at any rate at atmospheric pressure, the instability may be inhibited from developing as soon as sparking occurs. It was

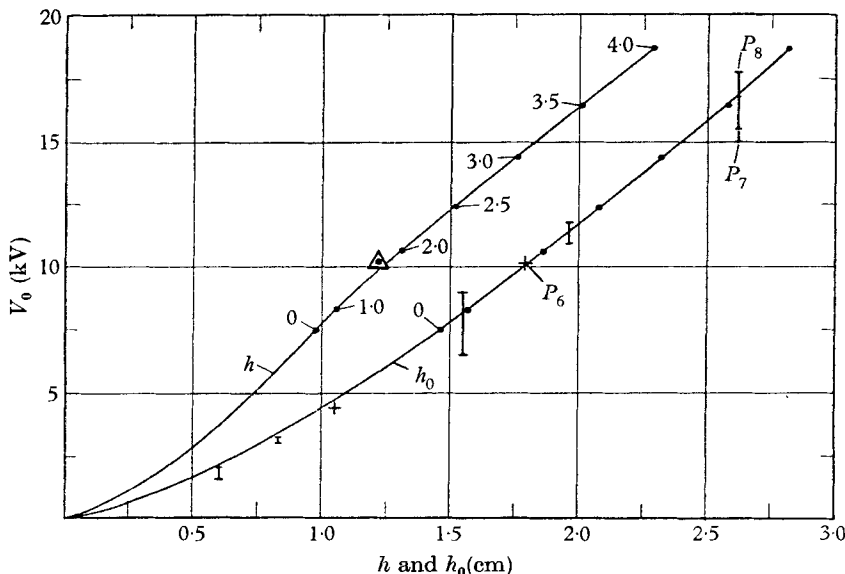


FIGURE 5. Voltage necessary for instability of oil-water interface. Full lines  $h$  and  $h_0$  are calculated. Vertical lines and crosses represent experimental results.  $P_6$ ,  $P_7$  and  $P_8$  represent values of  $h_0$  at which photographs figures 6, 7 and 8 (plate 1) were taken.  $\Delta$  is mean value of  $h$  derived from figure 6.

to study this instability under conditions where sparking would not be expected to modify the electric field till a later stage that the experiments with oil were undertaken. It was found more difficult to decide what stage in the development of the unstable disturbance corresponded most nearly with the calculated instability, because water bubbles in the oil or other irregularities in the oil-water interface were liable to give rise to disturbances which could become finite even when infinitesimal disturbances die down. For this reason there was often a range of values of  $V_0$  at which the interface first became disturbed. These are indicated in figure 5 where the spread of the observed values of  $V_0$  for a given value

of  $h_0$  is shown by vertical lines. The calculated values of  $h_0$  and  $V_0$  using the data of table 2 are given in columns 8 and 9 of table 1 and are shown in figure 5. The corresponding calculated values of  $h$  are also shown.

It will be noticed that the spread of values of  $V_0$  at  $h_0 = 0.61, 0.86, 1.05$  cm is small but at  $h_0 = 1.55, 1.96$  and  $2.62$  cm it is large. This is probably because the interfacial tension is rather an indefinite quantity. Below the value of  $h_0$  corresponding with  $h' = \sqrt{3}$ , the lowest value of  $V_0$  for instability corresponds with infinitely long waves ( $kh = 0$ ). The calculated value of  $V_0$  at which  $h' = \sqrt{3}$  is  $7.5$  kV (column 9, table 1). This is marked in figure 5 on the curves for  $h$  and  $h_0$  by the symbol 0. Other values of  $kh$  are marked  $1.0, 2.0, 2.5, 3.0, 3.5, 4.0$  in figure 5. Below  $V_0 = 7.5$  kV the instability depends only on  $\rho_1 - \rho_2$ , but above that value the surface tension begins to be important. With water under air the surface tension has a definite value but when the upper fluid is oil the surface tension is likely to vary if the scraper (shown in figure 3) is not used very frequently: nevertheless the upper limit of the observed values of  $V_0$  lie close to those calculated.

#### *Photographs of the water-oil interface*

Figures 6, 7, and 8 (plate 1) are photographs of the water-oil interface taken through the wall of the tank by a camera centred slightly above its undisturbed level but below that of the electrode. The illumination was by a translucent screen behind the tank and the white areas of the lower parts of the photographs are due to unobstructed light through the water. The upper edge of this area is the line where the interface intersects the side of the tank. It is slightly curved because the tank was only a few cm wider than the diameter of the electrode, and the part of the interface raised by the electric field before instability set in extended as far as the tank wall. The electrodes are seen near the tops of the photographs and the upper surface of the oil above them.

Figure 6 is a photograph of the oil-water interface when the agitation was beginning and  $h_0 = 1.9$  cm. Though the level of the fluid at the onset of instability cannot be measured accurately it will be seen that it has risen about  $\frac{1}{3}H$  above the undisturbed level, as it should when  $kh = 0$ . The value of  $h$  obtained by drawing a horizontal line which lies between the crests and troughs on the central part of the field gave  $h$  as  $1.22$  cm. This is shown in figure 5 as a triangle. The observed value of  $V_0$  corresponding with figure 6 was  $10.1$  kV which is very close to the calculated value.

Figure 7 shows what sometimes occurred at values of  $V_0$  below that corresponding with violent agitation and the point  $P_7$  in figure 5 represents this condition. A single cone, possibly caused by some accidental disturbance would move about over the interface. In this case  $h_0$  was  $2.62$  cm and  $V_0$  was about  $15$  kV. On raising the voltage to  $17.7$  kV,  $P_8$  in figure 5, the surface became violently agitated and the photograph shown in figure 8 was taken. It will be noticed that there are a large number of pointed crests when  $h_0 = 2.62$  cm. There were never more than one or two when  $h_0 = 1.79$  cm. This seems to be in qualitative agreement with the fact that the calculated value of  $kh$  for the photograph of figure 6 is  $1.8$  while that corresponding with figure 7 is  $3.5$ .

*Oil-dilute alcohol interface*

The reduction in density difference between the two fluids from 0.118 to 0.042 made the instability occur at lower potential gradients but the reduction in surface tension did not make it possible to attain higher values of  $kh$  than were attained with the oil-water interface. This was due to the limitation imposed by the size of the apparatus. At the highest value of  $kh$  used,  $V_0$  was under 9 kV but the ratio of  $h_0$  to the diameter of the electrode was not small enough to permit the use of the theoretical model which virtually assumes this ratio to be

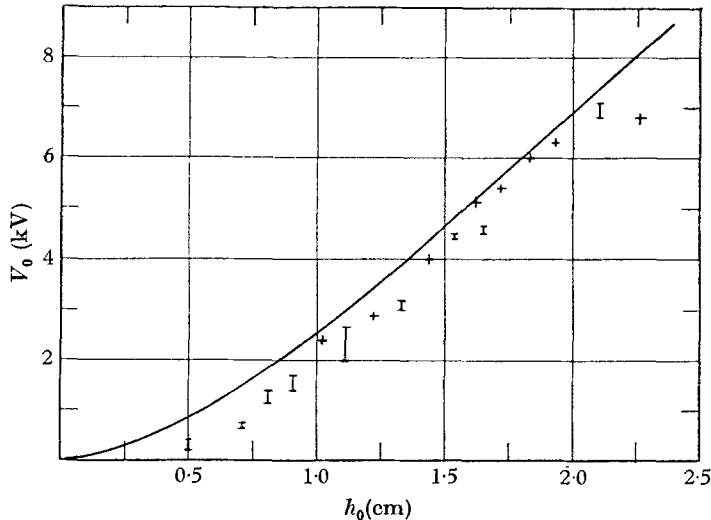


FIGURE 9. Interface between transformer oil and a mixture of equal volumes of water and alcohol. Line is theoretical value of  $h_0$  using  $\rho_1 - \rho_2 = 0.042$ ,  $T = 9.3$  dyn/cm.

negligible. Figure 9 shows the experimental results and the theoretical relationship calculated using the data of table 2. Corresponding calculated values of  $h_0$  and  $V_0$  are given in columns 10 and 11, table 1. These are shown in the curve of figure 9. It will be seen that except at the low values of  $h_0$  the experimental points are not far below the calculated curve. The reason for the large errors below  $h_0 = 1$  cm was not clear but it was noticed that the readings of the ammeter were often large and varying so that the correction given in (22) was rapidly varying and often nearly as large as the reading of the voltmeter. This large current might have been due to a residue left in the oil after the collapse of a jet. Contamination of the oil in the presence of alcohol no doubt contributed to residue formation.

*Experiments with mercury as the conducting fluid*

No unstable disturbance was produced when a shallow circular trough 8.1 cm diameter filled with mercury was placed under the upper electrode (12.7 cm diameter) and the potential raised till through the air sparking occurred. The sparks were always on the mercury meniscus at the rim of the trough, no doubt

because of the concentration of electric field there. The tank was then filled with oil and the potential of the upper electrode raised. The mercury became agitated at the rim and jets of mercury came off it. To make a comparison with the theory it was necessary to avoid electric stress concentration at the curved edge of the mercury. A steel disk 6.35 cm diameter and 6.5 mm thick with rounded edges was attached to the lower side of the electrode  $E_1$  (figure 3). This diameter being smaller than that of the mercury made it possible for the maximum electric stress at the mercury surface to be confined to the central region of its surface and in

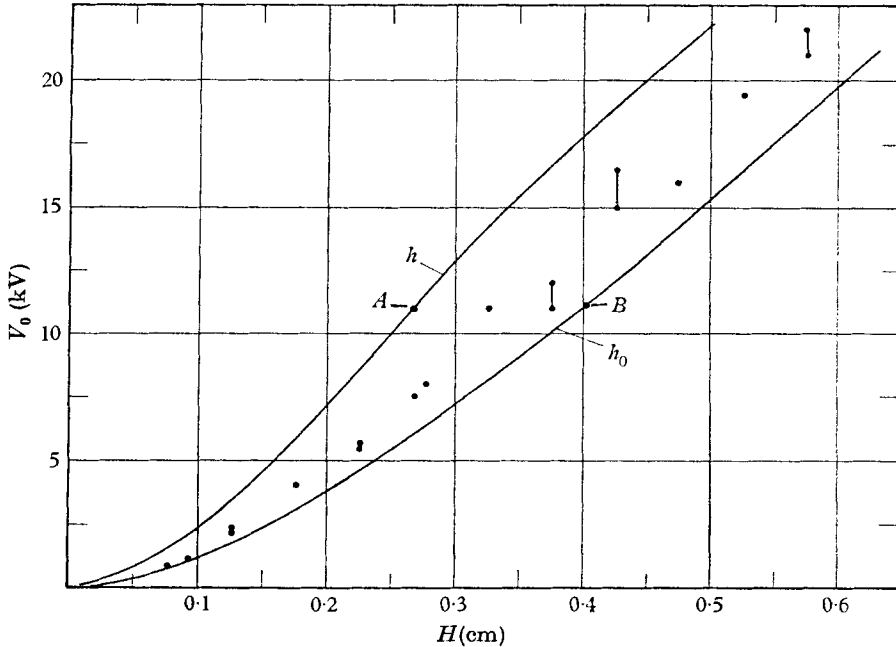


FIGURE 10. Experimental values of  $H$ , transformer oil-mercury interface.  
 $A$  and  $B$  calculated values of  $h$  and  $h_0$  for  $h' = \sqrt{3}$ .

fact when unstable points of mercury rose they were equally likely to appear at any point in the central region. After each experiment the mercury surface was scraped so that any film caused by the previous experiment would be removed.

It was found that as  $H$  the height of the steel electrode above the original height of the mercury increased definite and repeatable values of  $V_0$  at which mercury jets rose from the surface were found so long as  $H$  was less than 0.31 cm. The experimental points are plotted in figure 10.

The values of  $V_0$  and corresponding values of  $h$  and  $h_0$  were calculated from the data of table 2, and are given in columns 12, 13 and 14 of table 6. These are plotted in figure 10. It will be seen that the experimental values of  $H$  lie between them. Since the experimental arrangements made it impossible to measure  $\delta h$ , the drop in level at points outside the influence of the electric field experimental measures of  $h$  and  $h_0$  were impossible. It is satisfactory, however, that the experimental points lie between the calculated  $h$  and  $h_0$  curves. The value of  $V_0$

at which  $h' = \sqrt{3}$  is 11 kV. Repeatable experimental results were obtained when  $H$  was less than 0.3 cm and  $V_0 < 11$  kV, but when  $H > 0.3$  cm the measured critical values of  $V_0$  were variable. This is indicated in figure 10 by the spread of points at  $H = 0.34, 0.41$  and  $0.54$  cm.

#### 4. Conclusions

It has been shown theoretically that horizontal interfaces between conducting and non-conducting fluids become unstable under the action of a sufficiently great electric field. Experiments to verify the theory have revealed that electric failure of the non-conductor can arise in two manners. In the first, the non-conductor suffers the normal form of breakdown (occurring between solid electrodes, for example) at a voltage gradient lower than that which will cause instability of the interface. Such a situation arises in air over mercury. In the second, instability precedes dielectric breakdown, and the conducting jets formed from the unstable disturbances precipitate the failure of the non-conductor.

The theory for the stability of the interface does not involve a detailed knowledge of the form of the unstable disturbance. Nevertheless, in experiments the onset of instability was predicted generally with good accuracy.

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